The Mathematics of Neural Networks: One year later

September 2023—August 2024

Long-term outcomes report for the Mathematics of Neural Networks research theme of the Western Academy for Advanced Research

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Western Academy for Advanced Research

Contents

Photographs on cover page *(clockwise from top left): Visiting Fellow Alex Lubotzky (Weizmann Institute); Visiting Fellow Maria Chudnovsky (Princeton); Western Fellow Jan Mináč; group photo with (left to right) Contributing Researcher Jamie Graham, PDA Michal Cizek, Contributing Researchers Esther Yartey, Mohamad Kesserwan, Chris Steward, Gabriel Benigno, Erwan Martin, PDA Luisa Liboni, Theme Leader Lyle Muller, Contributing Researchers Aishwarya Pathak, Roberto Benigno, Alexandra Busch*

Successes at-a-glance

$1.5M

Grants
& awards

20+*

Associated publications since 2022

* Includes pre-print

Fields Lab for Network Science

Lasting Connections

The Mathematics of Neural Networks: One Year Later (Sept. 2023—Aug. 2024)

Key highlights:

- Use of stem cell technologies
- A new understanding of artificial intelligence
- Fields Lab for Network Science
- Building international partnerships

Neural networks (or "deep learning") could unravel the mysteries of the human brain. With the support of the Western Academy for Advanced Research (WAFAR), Western Fellows Lyle Muller (Brain and Mind Institute, Mathematics), Ján Mináč (Mathematics) and Marieke Mur (Psychology, Computer Science) were afforded the dedicated time, space, and funding to bring together an international team dedicated to exploring artificial neural networks and their possibilities. With their combined expertise in applied mathematics, computer science, and neuroscience, the Mathematics of Neural Networks research theme produced novel mathematical discoveries that could lead to greater understanding about neural functions and disorders.

During their tenure with WAFAR (September 2022—August 2023), the theme produced an impressive body of work that included groundbreaking insights into explainable artificial intelligence as well as exciting new developments about Rett Syndrome, a serious neurological disorder that mostly impacts young girls. Their excellence in research helped the theme secure a combined total of $1.5 million CAD, which is to be directed from major grants awarded by the BRAIN Initiative and National Eye Institute, under the U.S. National Institutes of Health.[1]

Such impressive outcomes would not have been possible without the strength of the team that included four international Western Academy Visiting Fellows, three WAFAR-funded post-docs,

[1]To be directed from an NIH BRAIN Initiative U01 (MPI) grant ($7,327,501) for 2023-2026, NIH NEI R01 grant (2,688,533) for 2023-2028.

and a substantial team of graduate-level contributing researchers. Theme Leader Lyle Muller credits WAFAR with providing the right support, at the right time, to help the team take their research to the next level:

> **"The collaborative environment fostered by the Western Academy has been key to our success, providing a fertile ground for ideas to flourish and collaborations to thrive. We look forward to building on this partnership, pushing the boundaries of knowledge in neural network theory, and contributing to the advancement of both artificial intelligence and neuroscience."**

In the year since the Mathematics of Neural Networks theme completed its official period of operations at WAFAR, it has made progress in four crucial areas of focus: 1.) using stem cell research to bridge the gap between mathematical ideas and medical solutions, 2.) forging new pathways in artificial intelligence research, 3.) establishing the Fields Lab for Network Science as a centre for neural network theory, and 4.) building and sustaining international partnerships.

1. Bringing the gap between mathematical ideas and medical solutions

Since its launch in September 2022, the Mathematics of Neural Networks theme has aimed to translate mathematical insights into "real-world" clinical interventions. Funding directed from NIH BRAIN initiative and NEI grants ($1.5 million CAD), has allowed the theme to harness the latest advancements in stem cell technologies. The funds will be put towards the use of a multi-electrode array to study neural activities in induced pluripotent stem cells (iPSC) models, thereby enhancing the theme's knowledge about the structure and function of neural connections.

A crucial partnership has emerged out of this project. Together with 2024 Allyn Taylor recipient Fred Gage (Salk Institute), the theme is continuing to build from the groundwork laid by their 2022 investigation into Rett Syndrome. Using neural network modelling, the researchers are working to create a computational framework for understanding neural circuits cultured from stem cells. The aim of this project is to foster a deeper understanding of neural networks and

what they might reveal to us about neurological functions and disorders. By drawing upon a wide range of interdisciplinary perspectives—ranging from mathematics, neuroscience, and stem cell biology-- the team aspires to solve important problems that may lead to the development of new treatments for patients with neurological diseases.

2. Towards a new understanding of Artificial Intelligence

Despite great advancements in artificial intelligence, developers still do not fully understand how AI makes decisions. This challenge is known as the "explainability" or "black box" problem. As AI technologies become more widespread, the explainability problem has drawn considerable attention from concerned critics. If poorly designed, or if used with malicious intent, AI systems could have dire consequences on privacy, human rights, and public trust. Advancements in AI explainability could help strengthen public trust by leading to safer and more transparent systems.

The Mathematics of Neural Networks theme has paved the way for new discoveries that could influence the design of future AI systems. During their productive tenure with WAFAR, the theme produced new crucial insights into how AI works. Along with researchers from the Salk Institute for Biological Studies, theme members developed a "first of its kind" technique that makes sense of how artificial neural networks generate predictions. Results from this project reached a wide readership and gained significant attention on social media, particularly on Twitter/X.[2]

In 2024, Muller and his team continued this line of research when they partnered with Todd Coleman (Stanford) to develop an explainable neural network. This breakthrough allows programmers to watch a neural network "in action" while a mathematical tool explains how it works. Significantly, this development demonstrates that certain computations *can* be fully explainable, a finding that could have larger repercussions in the study of neural networks and

[2] Results were published in Nature Communications, ranking in the 96th percentile of articles of a similar age as of September 2024. The article attracted considerable social media attention, ranking in the top 5% of all research outputs scored by Altmetric.

in AI research more broadly. Published in *Communications Physics* in July 2024, this study is part of a growing archive of impactful research. *See Appendix for a full publication list.*

3. The Fields Lab for Network Science

Launched in 2024 as part of the new Fields-Western Collaboration Centre at Western University, the Fields Lab for Network Science builds upon and promises to enhance the success of the Mathematics of Neural Networks theme. Created in partnership with the Fields Institute (University of Toronto), a prestigious centre for mathematical learning and innovation, the Fields Lab brings together leading thinkers in discrete mathematics, graph theory, number theory, and physics. Co-Director Lyle Muller recognizes the vital importance of this collaboration to the success of the project: “this partnership not only validates the significance of our work, but also provides a platform for broader dissemination and collaboration, potentially accelerating progress in this critical area of study.”

Under the leadership of Western Fellows Lyle Muller and Jan Mináč, the Fields Lab aims to spur cross-institutional connections between mathematicians and leading experts in neuroscience. Plans are underway for workshops and educational programming and residencies with distinguished visitors like Todd Coleman (Stanford) and John Reynolds (Salk Institute) that will establish the Fields Lab’s reputation as a global hub for neural network theory.

4. Engagement with international thinkers

The Mathematics of Neural Networks theme brings inspiring minds to Western. Globe-spanning collaborations with 2012 MacArthur Fellow Maria Chudnovsky (Princeton), visual processing expert Frederic Chavane (Centre National de la Recherche Scientifique), Israel Prize recipient Alex Lubotzky (Weizmann Institute), and acclaimed number theorist and mathematics educator Christian Maire (Franche-Comté Électronique Mécanique Thermique et Optique - Sciences et

Technologies), highlight the reach and impact of the theme's discoveries. Already the theme has leveraged these connections into sustained research partnerships that hold potential for future groundbreaking work. Currently, Lyle Muller and Frederic Chavane are developing potential projects for the Horizon Europe program, which could open new doors for international partnership. The Fields Lab for Network Science will continue to amplify the theme's international and cross-institutional connections by attracting leading scientific experts to Western's campus.

Future Directions

The Mathematics of Neural Networks theme aims to establish itself as an international centre for neural networks research. To make this goal a reality, the theme has identified the following priorities for the coming year (2024-2025):

- Building the theme's international reputation as trailblazers in the advancement of mathematical frameworks for understanding neural networks
- Leveraging their growing reputation and connections with the Fields Institute to foster new collaborations with other institutions and researchers.
- Drawing upon new research connections to uncover real-world applications for advancements in neural network theory
- Continuing to develop novel mathematical approaches to understand and explain neural networks
- Contributing new knowledge to the growing field of artificial intelligence and machine learning research
- Sharing findings through high-impact publications, conferences, and broader social channels to ensure their discoveries reach both academic and non-academic stakeholders

- Connecting their discoveries with humanity's most pressing issues in an era of accelerated technological change

By achieving these goals, the Mathematics of Neural Networks theme is well-positioned to establish itself as a key player in neural network research. According to Lyle Muller, this area of scientific discovery could shape the development and understanding of artificial intelligence technologies: "we anticipate that our work will contribute significantly to a topic that will certainly have a huge impact on human culture over the next five years." In doing so, The Mathematics of Neural Networks theme could put Western at the forefront of an exciting new field of inquiry whose significance will grow in the years to come.

> The collaborative environment fostered by the Western Academy has been key to our success, providing a fertile ground for ideas to flourish and collaborations to thrive.
>
> *Theme Leader Lyle Muller*

Appendix

Publication List

Benigno, G. B., Budzinski, R. C., Davis, Z. W., Reynolds, J. H., & Muller, L. (2023). Waves traveling over a map of visual space can ignite short-term predictions of sensory input. *Nature Communications*, *14*(1), 3409. https://doi.org/10.1038/s41467-023-39076-2

Botzanowski, B., Acerbo, E., Lehmann, S., Kearsley, S. L., Steiner, M., Neufield, E., Missey, F., Muller, L., Jirsa, V., Corneil, B. D., & Williamson, A. (2024). Focal control of non-invasive deep brain stimulation using multipolar temporal interface. *BioRxiv*.

Budzinski, R. C., Busch, A. N., Mestern, S., Martin, E., Liboni, L. H. B., Pasini, F. W., Mináč, J., Coleman, T., Inoue, W., & Muller, L. E. (2024). An exact mathematical description of computation with transient spatiotemporal dynamics in a complex-valued neural network. *Communications Physics*, *7*(1), 239. https://doi.org/10.1038/s42005-024-01728-0

Budzinski, R. C., Graham, J. W. C., Mináč, J., & Muller, L. E. (2023). Theory of transient chimeras in finite Sakaguchi-Kuramoto networks. *ArXiv*.

Budzinski, R. C., Nguyen, T. T., Benigno, G. B., Đoàn, J., Mináč, J., Sejnowski, T. J., & Muller, L. E. (2023). Analytical prediction of specific spatiotemporal patterns in nonlinear oscillator networks with distance-dependent time delays. *Physical Review Research*, *5*(1), 013159. https://doi.org/10.1103/PhysRevResearch.5.013159

Busch, A., Roussy, M., Luna, R., Leavitt, M. L., Mofrad, M. H., Gulli, R. A., Corrigan, B., Mináč, J., Sachs, A. J., Palaniyappan, L., Muller, L., & Martinez-Trujillo, J. C. (2024). Neuronal activation sequences in lateral prefrontal cortex encode visuospatial working memory during virtual navigation. *Nature Communications*, *15*(1), 4471. https://doi.org/10.1038/s41467-024-48664-9

Chebolu, S. K., Merzel, J. L., Mináč, J., Muller, L., Nguyen, T. T., Pasini, F. W., & Tân, N. D. (2023). On the joins of group rings. *Journal of Pure and Applied Algebra*, *227*(9), 107377. https://doi.org/10.1016/j.jpaa.2023.107377

Chudnovsky, M., Cizek, M., Crew, L., Mináč, J., Nguyen, T. T., Spirkl, S., & Tân, N. D. (2024). On prime Cayley graphs. *ArXiv*.

Davis, Z. W., Busch, A., Steward, C., Muller, L., & Reynolds, J. (2024). Horizontal cortical connections shape intrinsic traveling waves into feature-selective motifs that regulate perceptual sensitivity. *Cell Reports*, *43*(9), 114707. https://doi.org/10.1016/j.celrep.2024.114707

Davis, Z. W., Dotson, N. M., Franken, T. P., Muller, L., & Reynolds, J. H. (2023). Spike-phase coupling patterns reveal laminar identity in primate cortex. *ELife*, *12*. https://doi.org/10.7554/eLife.84512

Jain, P. B., Nguyen, T. T., Budzinski, R. C., Minac, J., & Muller, L. E. (2023). Synchronization patterns and stability of solutions in multiplex networks of nonlinear oscillators. *ArXiv*.

Keller, T. A., Muller, L., Sejnowski, T., & Welling, M. (2024). Traveling waves encode the recent past and enhance Sequence Learning. *ArXiv*.

Liboni, L. H. B., Budzinski, R. C., Busch, A. N., Löwe, S., Keller, T. A., Welling, M., & Muller, L. E. (2023). Image segmentation with traveling waves in an exactly solvable recurrent neural network. *ArXiv*.

Mináč, J., Muller, L., Nguyen, T. T., & Tân, N. D. (2024). On the Paley graph of a quadratic character. *Mathematica Slovaca*, *74*(3), 527–542. https://doi.org/10.1515/ms-2024-0040

Mok, R. S. F., Zhang, W., Sheikh, T. I., Pradeepan, K., Fernandes, I. R., DeJong, L. C., Benigno, G., Hildebrandt, M. R., Mufteev, M., Rodrigues, D. C., Wei, W., Piekna, A., Liu, J., Muotri, A. R., Vincent, J. B., Muller, L., Martinez-Trujillo, J., Salter, M. W., & Ellis, J. (2022). Wide spectrum of neuronal and network phenotypes in human stem cell-derived excitatory neurons with Rett syndrome-associated MECP2 mutations. *Translational Psychiatry*, *12*(1), 450. https://doi.org/10.1038/s41398-022-02216-1

Muller, L., Budzinski, R., Busch, A., Benigno, G., & Minac, J. (2023). Connecting structure of individual networks to dynamics in oscillator systems. *APS March Meeting Abstracts*.

Muller, L., Churchland, P. S., & Sejnowski, T. J. (2024). Transformers and Cortical Waves: Encoders for Pulling In Context Across Time. *ArXiv*.

Nguyen, T. T., Budzinski, R. C., Đoàn, J., Pasini, F. W., Mináč, J., & Muller, L. E. (2023). Equilibria in Kuramoto Oscillator Networks: An Algebraic Approach. *SIAM Journal on Applied Dynamical Systems*, *22*(2), 802–824. https://doi.org/10.1137/21M1457321

Nguyen, T. T., Budzinski, R. C., Pasini, F. W., Delabays, R., Mináč, J., & Muller, L. E. (2023). Broadcasting solutions on networked systems of phase oscillators. *Chaos, Solitons & Fractals*, *168*, 113–166. https://doi.org/10.1016/j.chaos.2023.113166

Pasini, F. W., Busch, A. N., Mináč, J., Padmanabhan, K., & Muller, L. (2023). Algebraic approach to spike-time neural codes in the hippocampus. *Physical Review E*, *108*(5), 054404. https://doi.org/10.1103/PhysRevE.108.054404

Piza, D. B., Corrigan, B. W., Gulli, R. A., Do Carmo, S., Cuello, A. C., Muller, L., & Martinez-Trujillo, J. (2024). Primacy of vision shapes behavioral strategies and neural substrates of spatial navigation in marmoset hippocampus. *Nature Communications*, *15*(1), 4053. https://doi.org/10.1038/s41467-024-48374-2

Rossi, K. L., Budzinski, R. C., Boaretto, B. R. R., Muller, L. E., & Feudel, U. (2023). Shifts in global network dynamics due to small changes at single nodes. *Physical Review Research*, *5*(1), 013220. https://doi.org/10.1103/PhysRevResearch.5.013220

Rossi, K. L., Budzinski, R. C., Medeiros, E. S., Boaretto, B. R. R., Muller, L., & Feudel, U. (2024). Dynamical properties and mechanisms of metastability: a perspective in neuroscience. *ArXiv*.

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